Spin Hall Current Driven by Quantum Interferences in Mesoscopic Rashba Rings

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We propose an all-electrical nanoscopic structure where *pure* spin current is induced in the transverse voltage probes attached to *quantum-coherent* one-dimensional ring when conventional unpolarized charge current is injected through its longitudinal leads. Tuning of the Rashba spin-orbit coupling in semiconductor heterostructure hosting the ring generates quasi-periodic oscillations of the predicted spin Hall current due to *spin-sensitive quantum-interference effects* caused by the difference in Aharonov-Casher phase acquired by opposite spins states traveling clockwise and counterclockwise. Its amplitude is comparable to the mesoscopic spin Hall current predicted for finite-size two-dimensional electron gases, while it gets reduced in wide two-dimensional or disordered rings.

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Introduction.—The increasing interest in spin-based information processing has fomented the field of *semiconductor spintronics* [1] where a plethora of concepts, exploiting fundamental quantum phenomena that involve electron spin, have arisen in order to generate and measure *pure spin currents*. In contrast to conventional charge currents or spin-polarized charge currents, which have been explored and utilized in metallic spintronics over the past two decades [2], pure spin currents emerge when equal number of spin-↑ and spin-↓ electron move in the opposite direction so that net charge current is zero [3]. Early [4] and recent [5, 6, 7, 8, 9] theoretical analysis has found potential sources of such current in: metallic or semiconductor paramagnets with spin-orbit (SO) dependent scattering on impurities (supporting extrinsic spin Hall effect [4, 5] as transverse spin current in response to longitudinal charge transport, or skew-scattering effects in Y-shaped semiconductor junctions [6]); multiprobe ferromagnet-normal metal hybrid devices [7]; optical injection in clean semiconductors [8]; and adiabatic spin pumping in mesoscopic systems [9]. Moreover, spin currents without accompanying charge current have been generated and detected in optical pump-probe experiments [10] and semiconductor quantum spin pumps [11].

Recent theoretical hints at the existence of *intrinsic* spin Hall effect in *clean* hole-doped [12] or electron-doped [13] semiconductor systems governed by SO couplings, where pure transverse spin current (substantially larger than in the case of extrinsic effect) is predicted as a response to longitudinal applied electric field, has attracted considerable attention. This is essentially a semiclassical effect in which current j_y^z of z-polarized spins flows along the y-axis within an infinite clean homogeneous semiconductor system penetrated by an external macroscopic electric field E_x along the x-axis. That is, it can be explained using a wave packet formalism [14] where current is generated by the anomalous velocity due to the Berry curvature of the Bloch states in SO coupled systems, rather than the displacement of the electron distribution function (as is the case of traditional charge currents accompanied by Joule heating). The generation and control of pure spin Hall current (that would be accompanied only by low-dissipative longitudinal charge current) could make possible spin manipulation without magnetic fields or problematic coupling of ferromagnetic electrodes to semiconductors devices [15].

The *non-equilibrium* spin current represents transport of spins between two locations in real space. However, intense theoretical striving to understand the nature of intrinsic spin Hall current, quantified by j_y^z [16] and spin Hall conductivity $\sigma_{sH} = j_y^z/E_x$, suggest that $j_y^z \neq 0$ might not imply real transport of spins since in dissipationless transport regime through a clean system it can be interpreted as an equilibrium background spin current existing even in the absence of any external electric field [17]. In addition, studies concerned with the influence of disorder (spin-independent scattering off static impurities) on spin Hall effect [18], as well as reexamination of the original arguments for clean systems [19], converge toward the conclusion that $\sigma_{sH} \rightarrow 0$ in an infinite homogeneous two-dimensional electron gas (2DEG) with Rashba SO interaction [20] (such SO coupling is pertinent to 2DEG since it stems from the inversion asymmetry of the quantum well confining electric potential). Nevertheless, quantum transport analysis of *measurable* [10, 21] spin-resolved charge currents $I_p^\uparrow$, $I_p^\downarrow$ and corresponding spin currents $I_p^s = \frac{\hbar}{2e}(I_p^\uparrow - I_p^\downarrow)$ in the ideal leads (without SO interaction) of multiprobe Hall bars accessible to experiments predicts that a type of spin Hall current will appear in the transverse voltage probes [21, 22, 23] attached to a finite-size 2DEG with Rashba SO interaction. This is due to the fact that spin currents in both the diffusive and the ballistic regime can be facilitated by macroscopic inhomogeneities [19]. Furthermore, possible signatures of spin Hall effect have been detected in finite-size 2D hole gases [24].

Thus, it becomes intriguing to pose two fundamental questions: Is it possible to induce spin Hall current in *strictly one-dimensional* systems with *no bulk*? Does quantum coherence (i.e., *spin-interference effects*) play any role in mesoscopic spin Hall current induction that

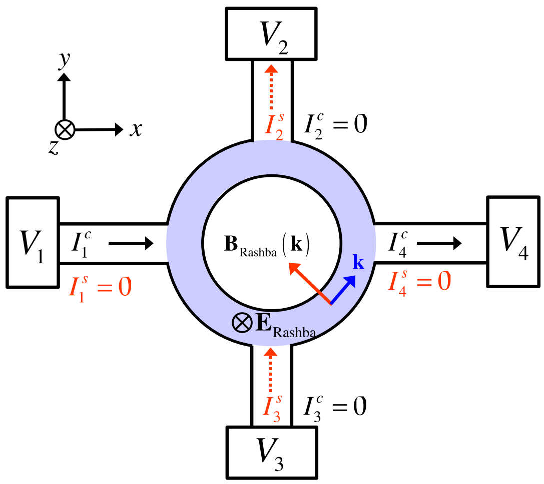

FIG. 1: The mesoscopic circuit serving as a generator of the pure ($I_2 = I_2^{\uparrow} + I_2^{\downarrow} = 0$) spin Hall current $I_2^s = \frac{\hbar}{2e}(I_2^{\uparrow} - I_2^{\downarrow}) = -I_3^s$ in the transverse voltage probes ($V_2 = V_3 \neq 0$, $I_2 = I_3 = 0$) attached to a ring realized using 2DEG in a semiconductor heterostructure [25]. The injected unpolarized ($I_1^s = 0$) current through (single-channel) longitudinal leads is subjected to the Rashba SO interaction (nonzero in the shaded ring region), which acts as a momentum-dependent pseudomagnetic field $\mathbf{B}_{\text{Rashba}}(\mathbf{k})$ arising due to the electric field $\mathbf{E}_{\text{Rashba}}$ confining the electrons to 2DEG.

can leave unique experimentally observable signatures? In this letter we undertake answering both of these questions by analyzing the spin-charge quantum transport in the presence of Rashba SO coupling within mesoscopic ring-shaped conductor (realized using 2DEG in semiconductor heterostructure [25]), which is modeled by the following single-particle effective mass Hamiltonian

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m^*} + \frac{\alpha}{\hbar}\left(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}\right)_z + V_{\text{conf}}(x,y) + V_{\text{dis}}(x,y). \quad (1)$$

Here $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli spin operator, $\hat{\mathbf{p}}$ is the momentum vector in 2D space, α is the strength of the Rashba SO coupling [20], and $V_{\text{conf}}(x,y)$ is the potential which confines electrons to a finite ring region. Such *Rashba ring*, attached to two longitudinal current probes and two transverse voltage probes (Fig. 1), will generate spin Hall current in the transverse leads. As demonstrated in Fig. 2 for 1D and in Fig. 3 for 2D rings, which are free of disorder $V_{\text{dis}}(x,y) = 0$, the spin Hall conductance $G_{sH}^z = I_2^s/(V_1 - V_4)$ measuring the magnitude of the transverse pure spin current in mesoscopic structures [21, 22, 23] will exhibit quasi-periodic oscillations, due to spin quantum-interference effects, when Rashba SO coupling is increased (e.g., via gate electrode covering the ring [26]).

The ring conductors smaller than the dephasing length $L_\phi \lesssim 1\mu$m (at low temperature $T \ll 1$K) have played an essential role in observing how *coherent superpositions* of quantum states (i.e., quantum-interference effects) on mesoscopic scale leave imprint on measurable transport properties. That is, they represent a solid state realization of a two-slit experiment—an electron entering the ring can propagate in two possible directions (clockwise and counterclockwise) where superpositions of corresponding quantum states are sensitive to the acquired topological phases in magnetic [Aharonov-Bohm (AB) effect] or electric [Aharonov-Casher (AC) ef-

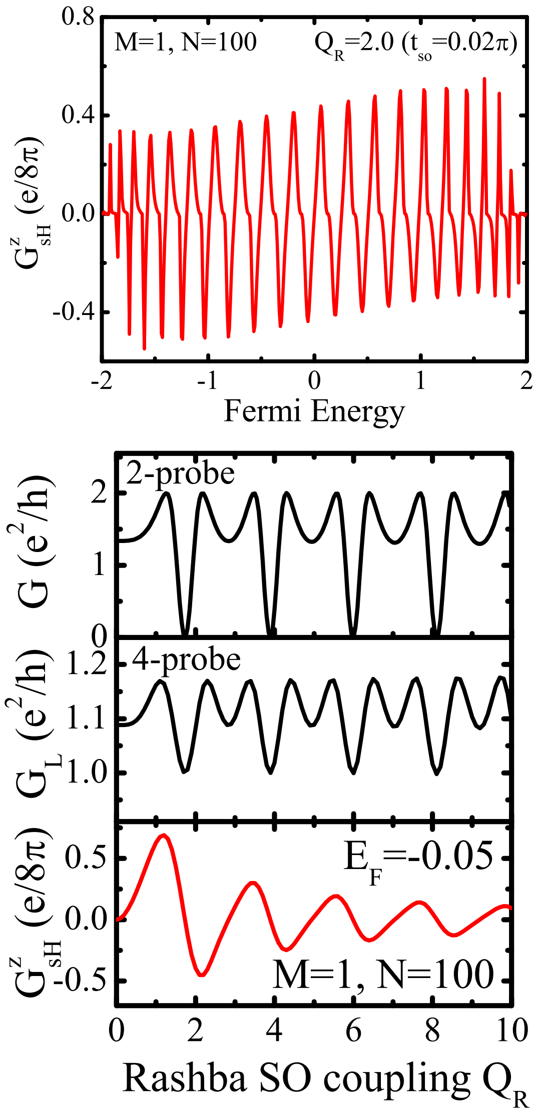

FIG. 2: The spin Hall conductance G_{sH}^z (corresponding to the detection of the z-component of pure spin current I_2^s) for 1D ring ($M = 1$, $N = 100$ lattice sites around the ring) attached to four single-channel leads as a function of the Fermi energy E_F (upper panel) and the dimensionless Rashba SO coupling $Q_{\text{R}} \equiv (\alpha/2at_0)N/\pi$ (lower panel). The lower panel also plots the charge conductance $G(Q_{\text{R}})$ of the corresponding two-terminal AC ring [28, 29] as well as the longitudinal charge conductance $G_L(Q_{\text{R}}) = I_4/(V_1 - V_4)$ of our four-terminal Rashba ring depicted in Fig. 1.

fect for particles with spin] external field whose changing generates an oscillatory pattern of the ring conductance [25]. Moreover, recently proposed *all-electrical* mesoscopic spintronic 1D ring device [27] would utilize the difference between AC phases of opposite spin states traveling clockwise and counterclockwise around the ring in a way in which their *spin interferences* will modulate the conductance of unpolarized charge current injected through single-channel leads between 0 and $2e^2/h$ by changing the Rashba electric field [28, 29].

Quantum transport of spin currents in 4-terminal Rashba rings.—The charge currents in mesoscopic structures attached to many leads are described by the multiprobe Landauer-Büttiker formulas [30]

$$I_p = \sum_{q \neq p} G_{pq}(V_p - V_q), \tag{2}$$

while the analogous formulas for the spin currents in the leads are straightforwardly extracted from them [6, 21]

$$I_p^s = \frac{\hbar}{2e} \sum_{q \neq p} (G_{qp}^{\rm out} V_p - G_{pq}^{\rm in} V_q). \tag{3}$$

Here $G_{pq}^{\rm in} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow} - G_{pq}^{\downarrow\downarrow}$ and $G_{qp}^{\rm out} = G_{qp}^{\uparrow\uparrow} + G_{qp}^{\downarrow\uparrow} - G_{qp}^{\uparrow\downarrow} - G_{qp}^{\downarrow\downarrow}$ have transparent physical interpretation: $\frac{\hbar}{2e} G_{qp}^{\rm out} V_p$ is the spin current flowing from the lead p with voltage V_p into other leads q whose voltages are V_q, while $\frac{\hbar}{2e} G_{pq}^{\rm in} V_q$ is the spin current flowing from the leads $q \neq p$ into the lead p (the standard charge conductance coefficients are expressed in terms of the spin-resolved conductances as $G_{pq} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} + G_{pq}^{\downarrow\uparrow} + G_{pq}^{\downarrow\downarrow}$ [31]). The linear response conductance coefficients are related to the transmission matrices $\mathbf{t}^{pq}$ between the leads p and q through the Landauer-type formula $G_{pq}^{\alpha\alpha'} = \frac{e^2}{h} \sum_{i,j=1}^{M_{\rm leads}} |\mathbf{t}_{ij,\alpha\alpha'}^{pq}|^2$, where $|\mathbf{t}_{ij,\alpha\alpha'}^{pq}|^2$ is the probability for spin-α' electron incident in lead q to be transmitted to lead p as spin-α electron and i, j label the transverse propagating modes (i.e., conducting channels) in the leads. The general expression for the spin Hall conductance is [21]

$$G_{sH} = \frac{\hbar}{2e} \left[(G_{12}^{\rm out} + G_{32}^{\rm out} + G_{42}^{\rm out}) \frac{V_2}{V_1} - G_{23}^{\rm in} \frac{V_3}{V_1} - G_{21}^{\rm in} \right], \tag{4}$$

where we choose the reference potential to be $V_4 = 0$. We emphasize that, in general, there are three non-zero spin conductances corresponding to three components of the polarization of transported spin [21]. For simplicity, we analyze only the z-component (i.e., we set the spin quantization axis for $\uparrow, \downarrow$ in Eq. (4) to be the z-axis).

We recall that Landauer transport paradigm spatially separates single-particle coherent and many-body inelastic processes by attaching the sample to huge electron reservoirs where, in order to simplify the scattering boundary conditions, semi-infinite ideal leads with vanishing spin and charge interactions are inserted between the reservoirs and the scattering region. Thus, even in the ballistic regime dissipation effects establishing steady state transport are always incorporated, in contrast to the artifacts of the Kubo formalism which maps the intrinsic spin Hall current in an infinite dissipationless system driven by the electric field to an equivalent system containing only equilibrium spin currents [17]. Here we clarify that apparent equilibrium solutions of the multiprobe spin current relations Eq. (3), $V_q = \text{const.} \Rightarrow I_p^s \neq 0$ found in Ref. [6] to originate from $G_{pq}^{\alpha\alpha'} \neq G_{pq}^{\alpha-\alpha'}$, actually *do not exist*. When all leads are at the same potential, a purely equilibrium non-zero term $\frac{\hbar}{2e}(G_{pp}^{\rm out} V_p - G_{pp}^{\rm in} V_p)$ (omitted in Ref. [6]) becomes relevant for I_p^s, canceling all other terms in Eq. (3) to ensure that no *unphysical* $I_p^s \neq 0$ would exist in the leads of an unbiased (V_q=const.) mesoscopic structure.

The stationary states of a system 1D ring + two 1D leads can be found exactly by matching the wave functions in the leads to the eigenstates of the ring Hamiltonian Eq. (1), thereby allowing one to obtain the charge conductance from the Landauer transmission formula [28]. However, attaching two extra leads in the transverse direction, as well the finite width of the ring and/or presence of disorder within the ring region, requires to switch from wave function to some type of Green function formalism. Here we employ the real⊗spin space Green function technique [21, 31] which yields the *exact* (within single-particle picture) transmission matrices $\mathbf{t}^{pq}$ between the leads p and q. The computation of the non-perturbative retarded Green function can be done efficiently using a local orbital basis representation of the Hamiltonian Eq. (1), which we have introduced in Ref. [29] as a set of M concentric chains composed of N lattice sites spaced at a distance a. The characteristic energy scales of such lattice Hamiltonian are: the hopping between neighboring sites $t_0 = \hbar^2/(2m^* a^2)$ (all energies will be measured in the units of t_0), and the Rashba hopping $t_{\rm so} = \alpha/2a$. It is also useful to measure the strength of the Rashba SO coupling within the ring region using a dimensionless parameter $Q_{\rm R} \equiv (t_{\rm so}/t_0)N/\pi$ [28, 29]. Since the contact between the ring and the leads can be controlled precisely using a quantum point contact to ensure that unpolarized current is injected through a single open conducting channel, we assume 1D electrodes ($M_{\rm leads} = 1$) while allowing for both strictly 1D rings $M = 1$ and 2D rings of finite width $M > 1$ [29].

Spin-interference effects in spin-Hall conductance.—The rapid oscillations of $G_{sH}^z(E_F)$ in Fig. 2 arise due to discrete nature of the energy spectrum in an isolated ring (note that once the leads are attached these eigenlevels acquire a finite width since electrons spend finite time inside the ring before escaping into the leads). The charge conductance of the two-probe 1D AC ring [27, 28, 29] becomes zero at specific values of $Q_{\rm R}^{\rm min}$ for which *destructive* spin-interference of opposite spins traveling in opposite directions around the ring takes place. For example, in a

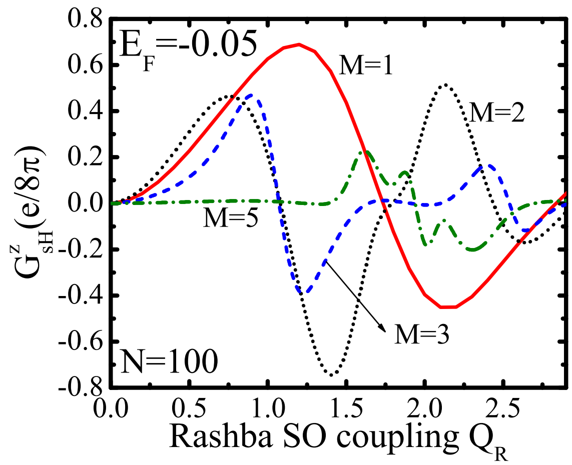

FIG. 3: The modulation of spin Hall conductance $G^z_{\rm sH}$ by changing the Rashba SO coupling $Q_{\rm R} \equiv (\alpha/2at_0)N/\pi$ in 2D ballistic rings of finite width (modeled by $M \geq 1$ coupled concentric 1D ring chains of $N = 100$ lattice sites) attached to four single-channel leads. The unpolarized current injected through the longitudinal leads is composed of spin-↑ and spin-↓ electrons at the Fermi energy $E_{\rm F} = -0.05$.

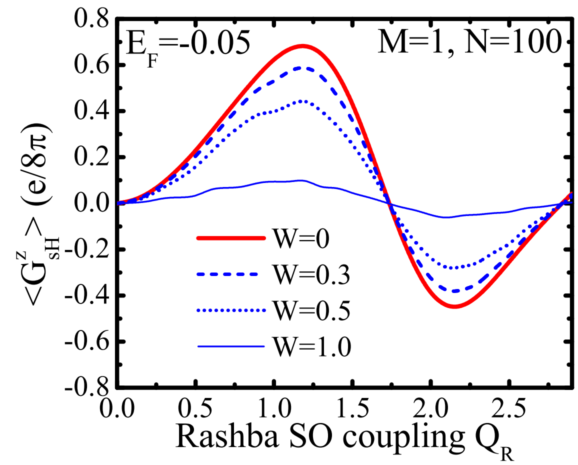

FIG. 4: The decay of the amplitude of disorder-averaged spin Hall conductance $\langle G^z_{\rm sH}\rangle$ with increasing strength W of the disorder introduced in the same 1D ring whose ballistic transport regime is examined in Fig. 2.

simplified treatment [28] $G = \frac{e^2}{h}[1+\cos(\Phi^{\uparrow}_{\rm AC} - \Phi^{\downarrow}_{\rm AC})/2]$, where $\Phi^{\sigma}_{\rm AC} = \pi(1+\sigma\sqrt{Q_{\rm R}^2+1})$ is the AC phase acquired by a spin-↑ or spin-↓ electron ($\sigma = \pm$ for ↑, ↓), has minima $G(Q_{\rm R}^{\rm min}) = 0$ at $Q_{\rm R}^{\rm min} \simeq \sqrt{n^2-1}$ ($n = 2, 3, 4, \cdots$). However, adding two extra transverse voltage probes onto the same 1D ring lifts the minima of the longitudinal conductance to $G_L(Q_{\rm R}^{\rm min}) = I_4/(V_1 - V_4) = e^2/h$. Nevertheless, the spin Hall conductance vanishes $G^z_{sH}(Q_{\rm R}^{\rm min}) \equiv 0$ at exactly these values of the SO coupling, while the amplitude of its quasiperiodic oscillations (which are not present in quantum spin-charge transport through simply-connected geometries [21]) gradually decreases at large $Q_{\rm R}$ due to reflection at the ring-lead interface [31].

Finally, we examine the observability of spin Hall current in realistic rings of finite width and in the presence of spin-independent impurities [18]. Figure 3 demonstrates that in 2D rings attached to four single channel probes the distinctive signatures—$G^z_{sH}(Q_{\rm R}^{\rm min}) = 0$ at specifically tuned (but harder to interpret [29]) $Q_{\rm R}^{\rm min}$—of quantum-interference dominated mesoscopic spin Hall effect can survive. When $M = 2$, we observe that the frequency of $G^z_{sH}(Q_{\rm R})$ oscillations is almost doubled. This is due to the presence of the second harmonics in the ring, which is a well-known effect in the AB rings with large radius/width ratio [32]. At larger widths, the quasi-periodicity of the $G_{\rm sH}(Q_{\rm R})$ is destroyed since accumulated AC phases average over many Feynman paths through the ring, thereby "dephasing" visibility of spin-interference effects [29]. When spin-independent scattering of static impurities occurs in disordered 1D rings, the amplitude of $G^z_{sH}(Q_{\rm R})$ in Fig. 4 is reduced with increasing disorder strength W of $V_{\rm dis} \neq 0$ in Eq. (1), simulated here by introducing a uniform random variable $\varepsilon_{\mathbf{m}} \in [-W/2, W/2]$ at each lattice site $\mathbf{m}$.

Conclusion—We predict that pure spin Hall current dominated by *quantum-interference* effects will be generated in mesoscopic ring-shaped 1D and 2D conductors and, in principle, could be observed by measuring its *unequivocal* experimental signature—quasi-oscillatory pattern of the SO coupling dependent voltage [3, 23] induced by the spin flow exiting from the Rashba spin-split *multiply-connected* region through the single-open-channel electrodes.

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[16] Note that j_y^z is the expectation value of an *ill-defined* quantum-mechanical operator in the presence of SO interaction since the plausible spin current operator $\hat{j}_i^k = \frac{\hbar}{4}(\hat{\sigma}_k \hat{v}_i + \hat{v}_i \hat{\sigma}_k)$ (symmetrized product of the Pauli matrices and the velocity operator) does not satisfy the continuity equation with the spin density operator due to the fact that spin is not conserved in SO coupled systems.
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